Meeting 6 Math 22

Reading Debrief:

- Discuss Section 10.1 w/ your group.
- Are there any questions you want me to address?

Questions?

- b(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

Path 1: Approach along x-axis
Parameterize: $r(t) = (t, 0)$, $t \geq 0$

Along Path 1: $\displaystyle\lim_{t\to 0} \frac{t\cdot 0^4}{t^2+0^8} = \lim_{t\to 0} 0 = 0.$

Path 2: The curve $r(t) = (t, t^{1/4})$, $t \geq 0$

Along Path 2: $\displaystyle\lim_{t\to 0} \frac{t\cdot(t^{1/4})^4}{t^2+(t^{1/4})^8} = \lim_{t\to 0} \frac{t^2}{t^2+t^2} = \lim_{t\to 0} \frac{1}{2} = \frac{1}{2}.$

Different limits along different paths ⇒ limit DNE.

Section 10.2 First-Order Partial Derivatives

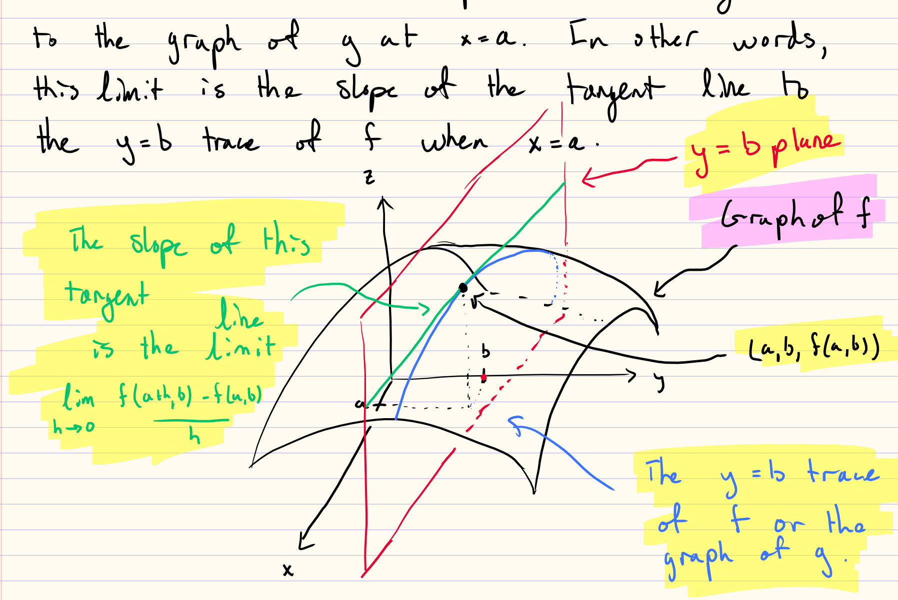
Let $f(x,y)$ be a function and (a,b) a point. There is a single variable function

$$g(x) = f(x, b)$$

obtained by "letting y be fixed". The graph of g is the $y=b$ trace of f. Since g is a single variable function, we can take a derivative

$$g'(a) = \lim_{h\to 0} \frac{g(a+h)-g(a)}{h} = \lim_{h\to 0} \frac{f(a+h,b)-f(a,b)}{h}.$$

This limit measures the slope of the tangent line to the graph of g at $x=a$. In other words, this limit is the slope of the tangent line to the $y=b$ trace of f when $x=a$.



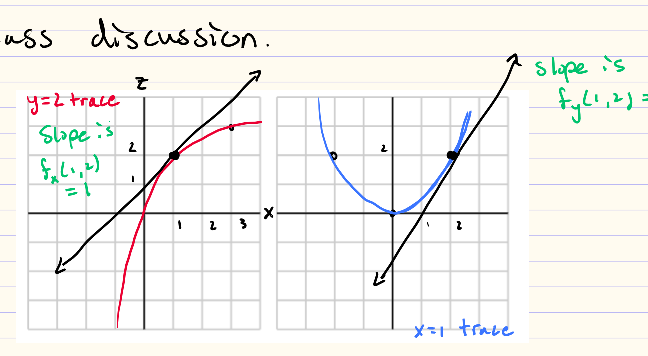
Definition 10.2.4 The first order partial derivatives of f with respect to x and y at a point (a,b) are defined via

- $\displaystyle\frac{\partial f}{\partial x}(a,b) = f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b)-f(a,b)}{h}$
- $\displaystyle\frac{\partial f}{\partial y}(a,b) = f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h)-f(a,b)}{h}$

when these limits exist.

Activity 10.2.2

- Complete Activity 10.2.2 and discuss w/ your group.
- Class discussion.



b. $\displaystyle f_x(1,2) = \lim_{h\to 0} \frac{f(1+h,2)-f(1,2)}{h}$ $\qquad f(x,y) = \frac{xy^2}{x+1}$

$\displaystyle = \lim_{h\to 0} \frac{\frac{4(1+h)}{2+h} - 2\cdot\frac{(2+h)}{(2+h)}}{h}$

$\displaystyle = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(2+h)} = \lim_{h\to 0} \frac{2}{2+h} = 1.$

d. $f_y(1,2) = 2$

As seen earlier, partial derivatives are just ordinary derivatives of an appropriate trace, which is just a single variable function. Therefore, all differentiation rules from Calc I apply!

Example

- $f(x,y) = 3x^3 - 2x^2y^5$
  $f_x(x,y) = 9x^2 - 4xy^5$
  $f_y(x,y) = 0 - 10x^2y^4$
- $g(r,s) = rs\cos r$
  $g_r(r,s) = s(1\cdot\cos r - r\sin r)$
  $g_s(r,s) = r\cos r$
- $f(w,x,y) = (6w+1)\cos(3x^2+4xy^3+y)$
  $f_w(w,x,y) = 6\cos(3x^2+4xy^3+y)$
  $f_x(w,x,y) = -(6w+1)\sin(3x^2+4xy^3+y)\cdot(6x+4y^3+0)$

Section 10.2.2 Intepretations of Partial Derivatives

The goal of the next activity is to interpret partial derivatives as rates of change in a physical context.

Activity 10.2.4

- Complete Activity 10.2.4 and discuss w/ your group.
- Class discussion.

a. Units of

C_T : $m/s\cdot{}^\circ C$, speed of sound relative to temp

C_S : $\frac{m\cdot L}{s\cdot g}$, speed of sound relative to salinity

C_D : $(m/s)/m = 1/s = hz$, frequency

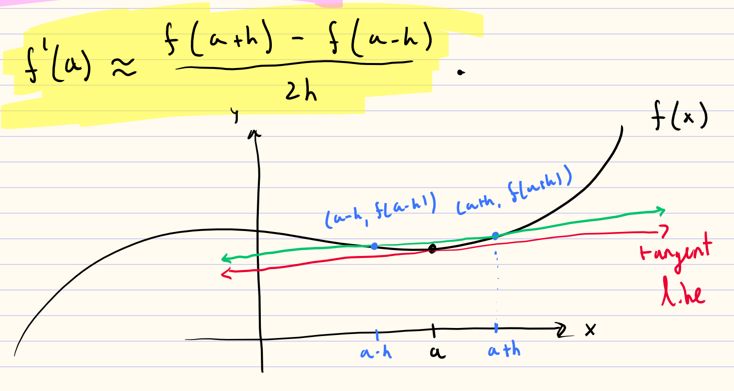
b. $C_T = 0 + 4.6 - 2\cdot(.055)T + 3\cdot(.00029)T^2 - (S-35)\cdot(.01)$

c. $C_T(10, 35, 100) = 3.587$
$C_S(10, 35, 100) = 1.24$
$C_D(10, 35, 100) = 0.016$.

Section 10.2.3 Estimating Partial Derivatives

The derivative $f'(a)$ of a single-variable function $f(x)$ at $x=a$ can be estimated using the "symmetric difference":

$$f'(a) \approx \frac{f(a+h)-f(a-h)}{2h}.$$



The slope of the green line is: $\displaystyle\frac{f(a+h)-f(a-h)}{a+h-(a-h)}.$

Activity 10.2.5

- Complete Activity 10.2.5 and discuss w/ your group.
- Class Discussion

a. Estimate $w_v(20,-10)$

$\displaystyle w_v(20,-10) \approx \frac{w(25,-10) - w(15,-10)}{2\cdot 5} = -\frac{1}{2}$

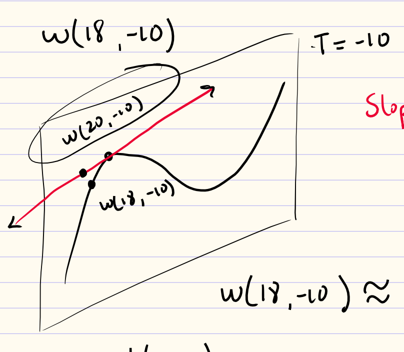
Units: ${}^\circ F/(m/h) = \frac{{}^\circ F\cdot h}{m}$ Wind chill relative to wind speed

b. Estimate $w_T(20,-10)$

$\displaystyle w_T(20,-10) = \frac{w(20,-5) - w(20,-15)}{2\cdot 5} = 13/10.$

Units: ${}^\circ F/{}^\circ F$, no units?

c. Estimate $w(18,-10)$



Slope: $w_v(20,-10)$

$h = -2$

$w(18,-10) \approx w(20,-10) + (-2)w_v(20,-10)$

$(f(x+h) \approx f(x) + hf'(x))$

$= -35 - 2\cdot(-1/2)$
$= -34$

d. Estimate $w(20,-12) \approx w(20,-10) + (-2)w_T(20,-10)$
$= -35 + (-2)\,13/10$
$= -37.6.$

e. Estimate $w(18,-12)$. How? My first idea is to use a tangent plane. Let's save this for Friday!